Math 163
Lab 9

Implicit differentiation and Related Rates

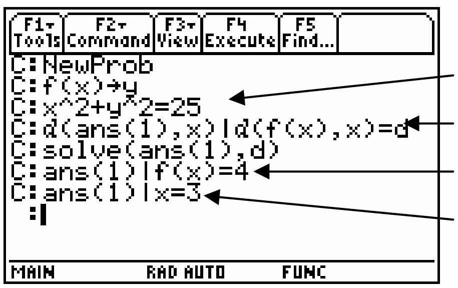
Consider the **implicit differentiation** problem:

Find $\frac{dy}{dx}$ at the point (3, 4): $x^2 + y^2 = 25$ Remember that $\frac{dy}{dx}$ is another name for *y'*

Of course, this is easy:

$$2x + 2y\frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{x}{y} = -\frac{3}{4}$$

Here is a script, **impldiff**, to perform this task:



original. equation.

name derivative d.

y-coord of pt

x-coord of pt

If you enter this script by typing the commands at the command line and then saving the script, wherever **ans(1)** appears, it is replaced with the result of the previous command. Therefore, *if you type this at the home screen by hand, and save it as a script,* then when you display the script for the first time in the text editor, you will have to change the lines 4, 6, and 7 back to what is shown at the left. This will not be an issue if your have downloaded the script.

Using the script: This script finds $\frac{dy}{dx}$ and then substitutes *x* and *y* values to find the numerical value of $\frac{dy}{dx}$.

If you just want to find $\frac{dy}{dx}$ in general (not at a particular point as we have above), simply ignore the last two lines of output. Of course, values must still be present in the script. Normally there will be values for *x* and *y* ($= f(x)$) left over from a previous problem. Also remember that $f(x)$ is a synonym for *y*. So you should replace $f(x)$ in the calculator answer with *y* in your written answer.

What the commands in the script do:

Line 1: **NewProb** clears out old variables so nothing interferes with this script.

Line 2: **f(x)→y** is used because we assume *y* is a function of *x* whenever we do implicit differentiation.

Line 3: the equation you are to work with

Line 4: finds the derivative with respect to *x*, with **d(f(x),x) ,** i.e. $\frac{dy}{dx}$, being named d.

Line 5: solves the resulting equation for d, i.e: it finds $\frac{dy}{dx}$. y will appear as **f(x)** in the result.

Lines 6 and 7: substitute values for *y* ($= f(x)$)and *x* and calculates the value of **d** ($\frac{dy}{dx}$). It is important to substitute for $f(x)$ first. If you substitute for *x* first, the resulting answer in the last line (**ans(1)**) would throw off that last calculation.

Now let's solve a **related rate** problem:

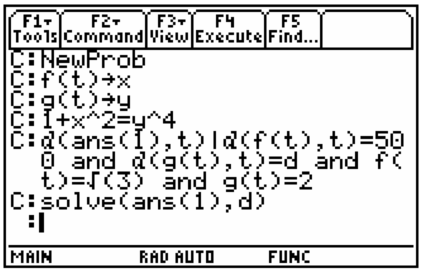
$$\text{Given: } 1+x^2=y^2, \text{ with } \frac{dx}{dt}=500, \quad x=\sqrt{3}, \quad y=2. \quad \text{Find } \frac{dy}{dt}.$$

Differentiating with respect to t: $2x\frac{dx}{dt}=2y\frac{dy}{dt}$

Substituting: $2\sqrt{3}(500)=2(2)\frac{dy}{dt}$

$$\therefore \quad \frac{dy}{dt}=250\sqrt{3}$$

The following **relrate** script will do this all for us:



When you enter the commands in the command line, **ans(1)** is replaced with the result of the previous command in the new command. This result is stored in the script. Therefore, *if you type at the home screen by hand, and save it as a script,* then when you display the script for the first time in the text editor, you will have to change the lines 5 and 6 back to what is shown at the left

The **NewProb** command clears out old variables so they do not interfere with the ones we will use.

f(t)→x and **g(t)→y** are used because when we differentiate with respect to *t*, we assume that *x* and *y* are functions of *t*.

> Keep in mind that x = **f(t)** and y = **g(t)**. Also, $\frac{dx}{dt}$ = ***d*(f(t),t)** and $\frac{dy}{dt}$ = ***d*(g(t),t)**.

The fourth command enters the equation we will differentiate with respect to *t*. Enter your equation here. It must be in *x* and *y*, so adjust the variables you use in the problem.

The fifth line of the script does all the stuff: We analyze it piece-by-piece.

***d*(ans(1),t)| *d*(f(t),t)=500 and *d*(g(t),t)=d and f(t)=√(3) and g(t)=2**

ans(1) refers to the answer to the previous calculation (the equation you entered).

***d*(ans(1),t)** differentiates both members of the equation in the fourth line with respect to **t**.

The "**|**" is the "with" operator, which makes the temporary substitutions specified.

The ***d*(f(t),t)=500** substitutes $\frac{dx}{dt}=500$. This was given in the original problem.

The ***d*(g(t),t))=d** stores $\frac{dy}{dt}$ in **d**. This is the quantity the problem requires us to solve for.

f(t)=√(3) and g(t)=2 are the last pieces of given information. **f(t)** is the x value, and **g(t)** is the y. These must be changed, along with the equation in line 4, when you use the script to solve a new problem.

The sixth line solves for **d** in the equation the line 5 generates. This is the missing value, in this case **d** = $\frac{dy}{dt}$.

CAUTION: You must make the appropriate entries in the "long line," depending on what information is given, and what you have to solve for. **d** is always used as the missing value which is to be determined.

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Lab 9 HOMEWORK

When you write the changed lines for both problems 1 and 2, *write them the way they appear in the script, not in ordinary mathematical notation.*

For example: $\dfrac{(x^2+y^2)^3}{(x-y)}$ would be written **((x^2+y^2)^3)/(x-y)**

1. Use **impldif** to find the slope of the tangent to the given curve at the specified point P. Show the changes in lines 3, 6, and 7 of your script for each problem.

a. $3(x^2+y^2)=100xy$; $P=(3,1)$ line 3______________________________

Line 6________ Line 7_________ Slope___________

b. $x^2(x^2+y^2)^2=y^2$; $P=(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})$ line 3____________________________

Line 6________ Line 7_________ Slope___________

c. $2(x^2+y^2)^2=25(x^2-y^2)$; $P=(3,1)$ (lemniscate)

line 3____

Line 6________ Line 7_________ Slope___________

2. Use **impldif** to find $\frac{dy}{dx}$ for each of the following curves. Show change in line 3 of your script for each.

a. $\sqrt{x^2y^2+1}=2xy$ Line 3______________________________ $\frac{dy}{dx}$________________

b. $\sin(x^2+y)=y^2(3x+1)$ Line 3______________________________ $\frac{dy}{dx}$________________

c. $\dfrac{y}{x-y}=x^2+1$ Line 3______________________________ $\frac{dy}{dx}$________________

d. $\sin x+\cos y=\sin x\cos y$ Line 3____

(may give message that solution may be incorrect.)

$\frac{dy}{dx}$____

e. Why do lines 6 and 7 of **script2** NOT matter in this question?

3. Use **relrate** to solve the following related rate problems. Show the changes in lines 4 and 5 of your script for each problem.

a. $y = x^3 + 2x$ and $\frac{dx}{dt} = 5$. Find $\frac{dy}{dt}$ when $x = 2$.

line 4 **C:**____

line 5 **C:*d*(ans(1),t)|*d*(f(t),t)=__________and *d*(g(t),t)= ____ and f(t)= ____ and g(t) =____**

answer____________________

b. $y = \sqrt{x^3 + 1}$ and $\frac{dy}{dt} = 4$ at the point $(2,3)$. Find $\frac{dx}{dt}$ at $(2,3)$.

line 4 **C:**____

line 5 **C:*d*(ans(1),t)|*d*(f(t),t)=__________and *d*(g(t),t)= ____ and f(t)= ____ and g(t) =____**

answer____________________

c. $y = 20\tan x$ and $\frac{dy}{dt} = 2$, find $\frac{dx}{dt}$ when $y = 15$ and $x = 10$.

line 4 **C:**____

line 5 **C:*d*(ans(1),t)|*d*(f(t),t)=__________and *d*(g(t),t)= ____ and f(t)= ____ and g(t) =____**

answer____________________

d. *If* $x^2 + xy - y^2 = 11$ and $\frac{dy}{dt} = 5$, find $\frac{dx}{dt}$ when $x = 4$ and $y = 10$.

line 4 **C:**____

line 5 **C:*d*(ans(1),t)|*d*(f(t),t)=__________and *d*(g(t),t)= ____ and f(t)= ____ and g(t) =____**

answer____________________