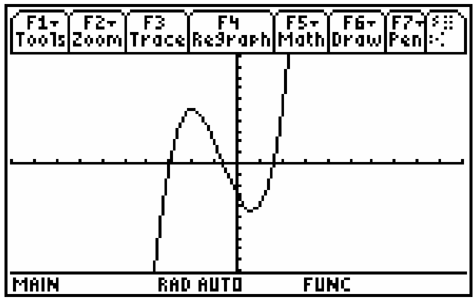
Math 163
Lab 4: Zeros, Extremes, Scripts and the Derivative

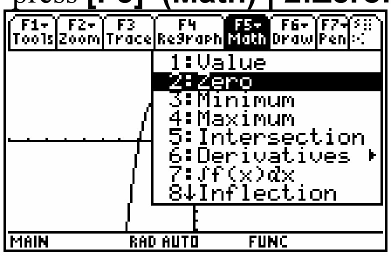
ZEROS

Graph **$Y1=x^3 + 2x^2 - 4x - 3$** in the standard viewing rectangle.

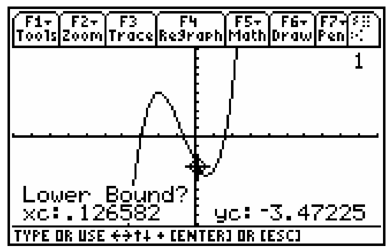


There are 3 x-intercepts (zeros) of this function from x = -10 to x = 10. We may find these from the graph window:

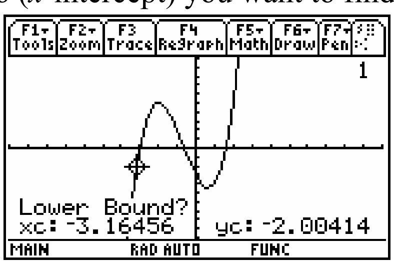
press **[F5] (Math) | 2:Zero.**



⇒

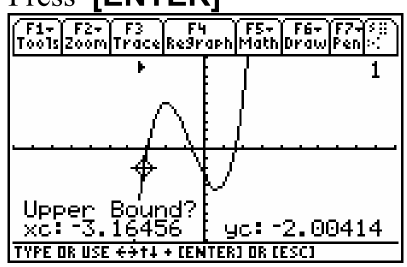


We will find the zero that is near -3.
Move the cursor a little to the left of
the zero (x-intercept) you want to find.

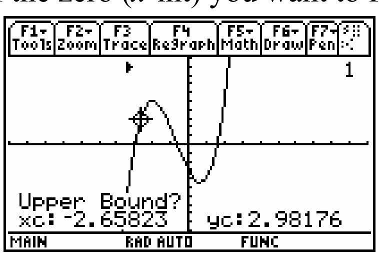


⇒

Press **[ENTER]**

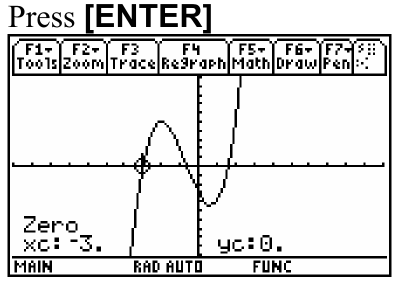


Move the cursor a little to the
right of the zero (x-int) you want to find.



⇒

Press **[ENTER]**



The value of the x-intercept, or zero, that is between the lower and upper bounds you chose, is calculated and displayed. Try this same process for each of the remaining zeros.

Other 2 zeros:_________, _________

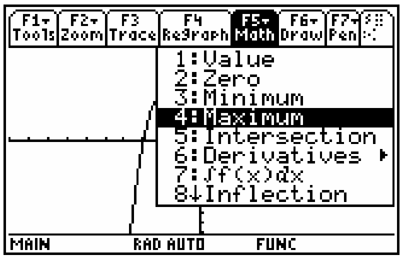
Correct to 3 decimal places, the three zeros are: - 3 (exact – the one we found above), also - 0.618 and 1.618. You may check these by going to the **[HOME]** screen, storing each value in *x*, then typing the expression $\boldsymbol{x^3 + 2x^2 - 4x - 3}$ and pressing **[ENTER]** to find its value.

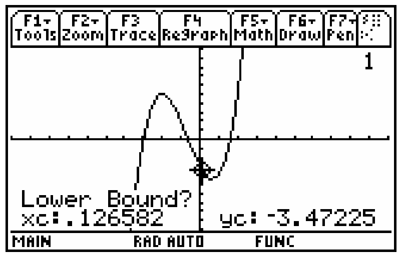
MAXIMUM and MINIMUM

Local Maximums and local minimums (high points and low points) on the graph are found in a similar way, using the **[F5] :Math** menu **, choices 3: minimum** and **4: maximum**.

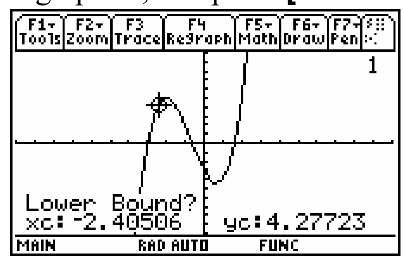
Let's find the local maximum of the graph above, using the calculator:

Press **[F5]:Math | 4:Maximum.**

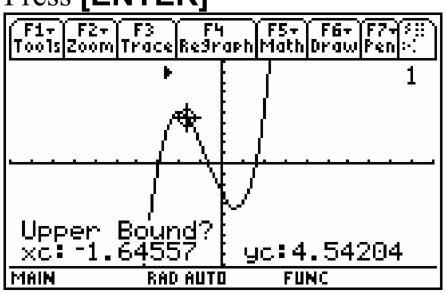




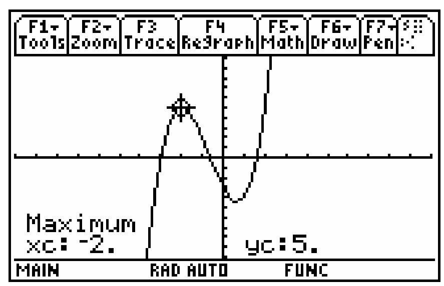
For lower bound, move the Cursor a little to the left of the high point, and press **[ENTER]**



For upper bound, move the cursor a little to the right of the high point.
Press **[ENTER]**



The high point (local maximum) is found, and the point and its Coordinates are displayed.



Finding a local minimum is done in a similar manner. Try it on the low point of the same graph.

Coordinates of the local minimum ________________

Scripts, the Difference Quotient, and the derivative at x = a

A Script is a series of stored commands that you could issue one at a time on the command line. It is not a program, in that it is not written in the programming language of the TI-89. If there is a series of commands that you would frequently use on the command line, it is a good candidate for a Script.

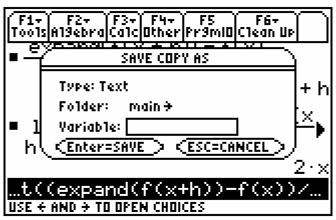
Let's write a script to do the symbolic work in finding a derivative.

Press **[CLEAR]**, then **[F1]** choice **8** to clear off the command line and the history area. Enter the following commands carefully, one at a time (press **[ENTER]** after each), on the command line:

Command to type	Explanation (do not type into calculator)
newProb	Erase old variables, etc.
Define f(x)= x^2	Defines the function to be used. This may be changed
4→a	Specify the value of *a* to be used. This may be changed
.01→h	Specify the value of *h*. This may be changed
expand(f(a+h))	Expand $f\,(a + h)$
expand(f(a+h)-f(a))	Numerator of the difference quotient (numerical)
expand(f(a+h)-f(a))/h	Display the difference quotient (numerical)
limit((expand(f(a+h))-f(a))/h,h,0)	The actual limit: the derivative at x = *a*.

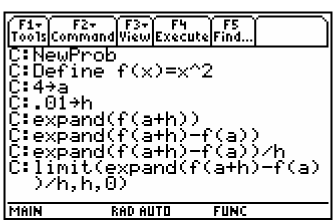
This series of commands will find the derivative of the function f using the definition, and should show enough of the individual steps for you to easily construct the complete steps in the work of finding of the derivative by definition

Press **[F1]: Choice 2:Save copy as**… This screen appears:

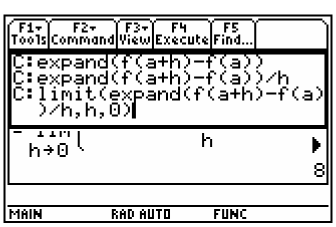


In the variable box, type **difquot** (this is short for difference quotient). Press **[ENTER]** twice. (Once to register the script name, the second time to register the changes made. You will be returned to the **HOME** screen.

On the TI-89, press **[APPS]-8: Text Editor**. (On the TI-89 Titanium Ed., Press **[APPS]** and use the cursor key to move to the **Text Editor** Choice; then press **[ENTER]**). On both calculators, choose option **2:Current**. Use the cursor keys to choose **difquot** if it is not already indicated. Your script is then displayed:



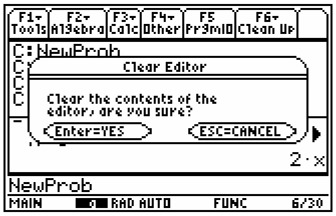
This is a listing of the commands you typed, each proceeded by **C**:. Press **F3: VIEW** and choose **option 1: Script View**:



This is a split screen. The script you created is at the top, and a portion of the home screen is at the bottom.

Use the cursor up key to move the cursor to the beginning of the first line of the script, before the "**N**" in "**NewProb**".

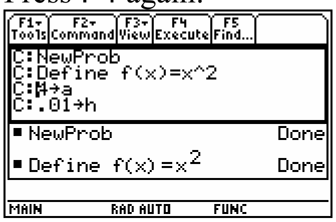
Press **F4**: to execute the next command in the script:



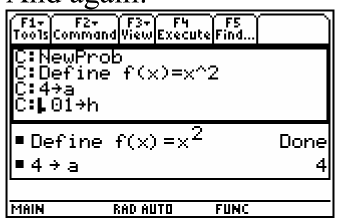
This **Clear Editor** screen may not appear. That is alright. It is just a warning. If it does appear, you must press **[ENTER]** to continue executing the steps of the script, in order.

The **NewProb** command is executed, just as if you had typed it at the keyboard, and you are asked if you are sure you want to continue. Press **[ENTER]** to clear the contents of the editor.

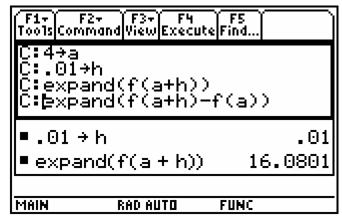
Press F4 again:



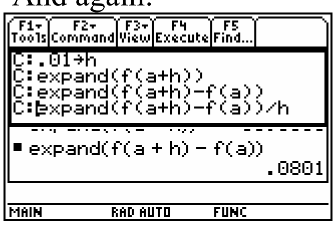
And again:



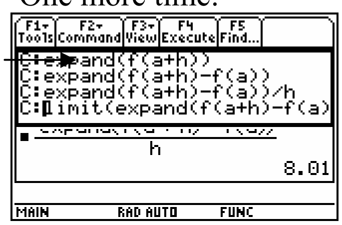
Two more times::



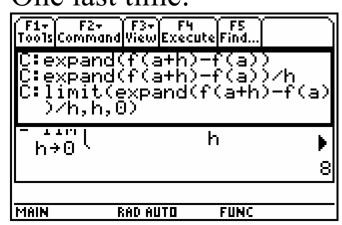
And again:



One more time:



One last time:



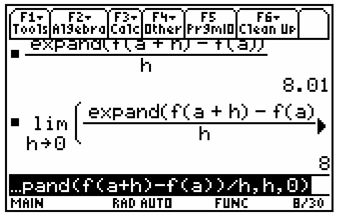
When the calculator is in split screen mode with the script in the top window, you may move the cursor around in the script by using the blue cursor keys. You may also modify any line of the script. (Save the new script if you wish.) Then move to the beginning of the **NewProb** line and execute the new script line by line by pressing the **F4** key

Press **[2nd] – [APPS]** ("Switch window") to move to the bottom half of the split screen. Then use the cursor keys to move around the screen. Notice that each command is listed there and carried out, in turn, just as if you had typed it.

Press **[2nd] – [APPS]** to return to the top half of the split screen. Move the cursor up to the **Define** line, and change **f(x)** to **f(x)=3x^2-2x**. Then move the cursor to the beginning of the **NewProb** line, and press **F4** repeatedly, as before. The steps in the solution to the derivative of the new function, using the definition of the derivative, are shown. If the display of any line on the bottom half of the screen extends past the right edge of the screen, you may the "Switch window" key (**[2nd]-[APPS]**) to move the cursor to the bottom half of the screen. Then use the cursor keys to move to the end of the display line you wish to see. Press "Switch window" (**[2nd]-[APPS]**) again to return to the top half of the split screen and continue executing commands from the script.

You may change the function to any you wish. If it involves a trig function, you must change the **expand** commands to **texpand** so that trig expansion will be done.

Press **F3: option 2: clear split**, then press **HOME**:



When you are done with this assignment, do not delete the script.
We will use it in a future lab activity/assignment.

* *

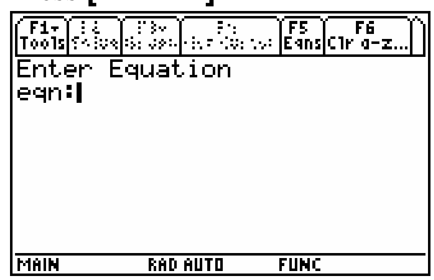
SUPPLEMENT: NUMERIC SOLVER (OPTIONAL)

The Numeric Solver is to be found in **[APPS]**, choice **9: Numerical Solver**.
(Press **[APPS] | 9:Numeric Solver,** or move the cursor down to choice **9**, and press **[ENTER]**)

[APPS]

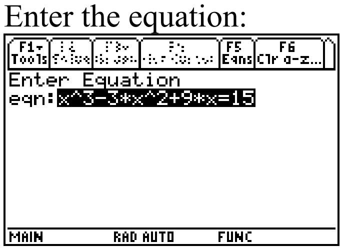
Move down the menu to choice **9**:

Press **[ENTER]**:

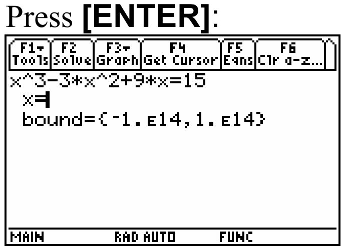


We will solve the equation: $x^3 - 3x^2 + 9x = 15$. Type the ***entire*** equation on the **eq:** line:

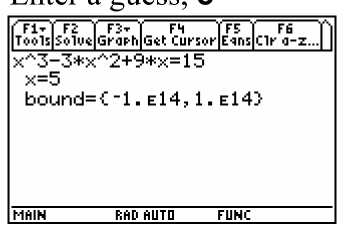
Enter the equation:



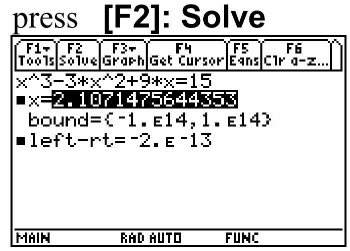
Press **[ENTER]**:



Enter a guess, **5**



press **[F2]: Solve**



The calculator will look for a root (zero) "close to" the value you pick, between the bounds you chose. Be sure the cursor is on the **x=** line when you press **[F2] :Solve**. If there are more real zeros, you would have to have some idea of the value of each to enter as your guess for the **x=** line, in order to be sure to find the one you are looking for, rather than some other real zero.

Check the solution graphically by graphing y1= $x^3 - 3x^2 + 9x - 15$ in the Standard Viewing Rectangle (Zoom | 6). Then use **[F5] :Math | 2:zero** as we did earlier to find the zeros graphically. They should agree with the values found using the numeric solver.

When using the Numeric Solver, there are a few things to keep in mind.

- Begin by pressing **[APPS] | 9:Numeric Solver**
- Several screens will appear in succession as you proceed.
- On the first screen, you enter the ***entire*** equation: For example: $x^2 - 3x - 8 = 0$
  On the **x=** line You would type: **x^2-3x-8=0**. (Be sure to remember the "= 0")
- Press **[ENTER]** , and the second screen shows. The equation you typed is displayed.
- On the next line, after **x=,** enter an initial guess. For this problem, use **x=5**.
- On the third line, the bounds are values of x between which the root you are looking for is located. They must be in curly brackets: **{ },** and separated by a comma. By default, the bounds shown are very small and very large numbers. Leave the bounds unchanged for the most part. This essentially does not limit the search among the real numbers – except for especially small negative numbers or large positive numbers.
- Move the cursor to the second line so that **x=5** is highlighted, and Press **[F2]: Solve**. (NOT **[ENTER]**!)
- The solution will be calculated and displayed. (**4.702**, to 3 decimal places)
- To find the other solution, move the cursor to the **x=** line, enter -2, (-2 is close to the other solution) and press **[F2] Solve.**
- The solution will be calculated and displayed. (**-1.702**, to 3 decimal places)

Notes:

- Although usually we leave the bounds unchanged, you may enter other values between the curly brackets to limit the search to a smaller interval.
- You need to have a reasonably good guess for the value you enter at the **x=** prompt on the second screen.

Use the Numeric Solver to find the real solution of the cubic equation $x^3 - 3x = -7$, to 3 decimal places. Fill in the four lines the screen shows when the solution has been determined:

equation:	______________________
x =	________
bound =	______________________
left-rt =	________

Lab #4 NAME(PRINT Clearly!)_____________________________Lab Time_______

1 – 2: Using the process outlined above on pages 1 and 2, sketch a graph and find the zeros and local maximum(s) or minimums (as directed) of the functions: (USE RADIAN MODE!)

- Be sure that there is only one zero, and there are no vertical asymptotes, between the lower and upper bounds you choose.
- Use **[F2] :Zoom**: | **6:zoomstandard** to view each graph initially. You may wish to zoom in on parts of the graphs to see things more clearly..
- Graph the function in function mode, and use the **[F5] :Math** menu choices zero, maximum, and minimum to find the zeros and local maximums or local minimums specified. Limit your searches and answers to the standard viewing rectangle (**Zoom | 6**)

1. $f(x) = 2x^2 - 6x - 3$ Graph:

Zero(s)_____________________

Local Min(s)_________________

2. $f(x) = \sqrt{x} - 2\sin x$

(Be sure Mode is **RADIAN)**

Graph in the standard viewing rectangle, and then zoom in around the origin to see additional x-intercepts, relative maximums and relative minimums.

Zero(s)_____________________

Local min(s)________________

Local max(s)________________

3. Use the script written in the lesson to find the difference quotient and derivative for each function given at $\boldsymbol{a} = 4$, $\boldsymbol{h} = .1$ and $\boldsymbol{a} = 4$, $\boldsymbol{h} = .01$. Use the table to record the individual output lines, beginning with the 5th line of the script: **expand(f(a+h))** The script will have to be modified slightly for each problem. Notice how the value of the difference function compares to the derivative at $\boldsymbol{a} = 4$. Set **[MODE]** to **float 6** and give all non-exact decimal values to 6 significant digits.

a) $x^4 - 3x^2 - 4$ b) $\sqrt{x-1}$

Script line	*a)* $f(x) = x^4 - 3x^2 - 4$		*b)* $f(x) = \sqrt{x-1}$	
--	$h = 0.1$	$h = 0.01$	$h = 0.1$	$h = 0.01$
expand (f(a+h))				
expand(f(a+h)-f(a))				
(*) **expand(f(a+h)-f(a))/h**				
(#) **limit((expand(f(a+h))-f(a))/h,h,0)**				
Difference between lines (*) and (#) (Subtract)				

4. The derivative we used in question 1 is sometimes called the "***one-sided derivative***", since the interval over which the change is approximated extends only on one side of the value of $\boldsymbol{a}$.

The ***SYMMETRIC DERIVATIVE*** is defined to be $f'(x) = \lim_{h \to 0} \frac{f(a+h) - f(a-h)}{2h}$. It is called the symmetric derivative because in order to approximate the slope at $\boldsymbol{a}$, it goes to points equally distant ($\boldsymbol{h}$ units) on both sides of the $\boldsymbol{a}$ value in question.

a) Complete the script below (fill in the blanks in column 1) that will find the symmetric derivative of a function. (It is only a slight modification of the script used above – refer to the table below. The blanks indicate where the changes must be made). Save and name your new script **symderiv**.

b) Run the new script for the functions in #1, again with $\boldsymbol{a} = 4$, $\boldsymbol{h} = .1$ and then again with $\boldsymbol{a} = 4$, $\boldsymbol{h} = .01$. Give nopn-exact decimal values to six significant digits.

Script line	*a)* $f(x) = x^4 - 3x^2 - 4$		*b)* $f(x) = \sqrt{x-1}$	
---	$h = 0.1$	$h = 0.01$	$h = 0.1$	$h = 0.01$
expand (f(a+h))				
expand(f(a+h)-f(a______))				
(*) **expand(f(a+h)-f(a_______))/(___))**				
(#) **limit((expand(f(a+h))-f(a______))/(__),h,0)**				
Difference between lines (*) and (#) (Subtract)				

5. Compare the "closeness" to the actual derivative in the symmetric derivative approximations in problem 4 to the one-sided derivatives in problem 3 (the last line in each of the two tables).

a) Which process gives a better approximation to the actual derivative – the one-sided derivative, or the symmetric derivative? Why? (Explain by referring to the graphs of the secant lines used to approximate the one-sided and two-sided derivative values.)

b) Why do you suppose we usually use the one-sided derivative when finding derivatives algebraically? i.e. Which is easier to do by hand?

6. a. Write a script named "taneq" that will write the equation of the tangent to the function $y = x^2 - 3x - 1$ at $x = 3$.
The lines of your script should perform the following tasks. Your script should have five lines – one for each of the five tasks shown:

- Start with the usual "**newProb**" command
- Define the function $f(x)$ to be $x^2 - 3x - 1$
- Store 3 in a
- Calculate $\lim\limits_{x \to a} \frac{f(x) - f(a)}{x - a}$ to find the derivative at $x = a$ and store the answer in m
- Solve the equation $y - f(a) = m(x - a)$ for y.

C: ____

C: ____

C: ____

C: ____

C: ____

b. Execute the script in part a to determine the equation of the tangent to the graph of $y = x^3 - x^2 + 3x - 5$ at $x = 1$.

6b.________________________

c. Graph the function and the tangent from part b on the same axes in the standard viewing rectangle.