Math 163

Lab 2

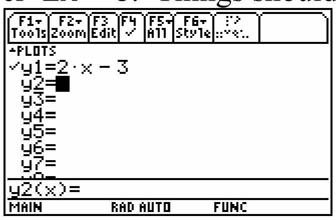
LIMITS

In Calculus we are frequently interested in questions like:

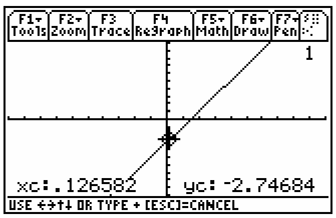
" *As x gets closer and closer to 5, <u>but not necessarily equal to 5</u>, what (if anything) does the value of the function f (x) = 2x – 3 get closer and closer to?*"

We may investigate this by examining the graph of **y1 = 2x – 3**.

1. Press **[♦]-[F1] to** get to the **[Y=]** screen
2. For **y1 =,** enter **2x – 3**. Things should now look like:

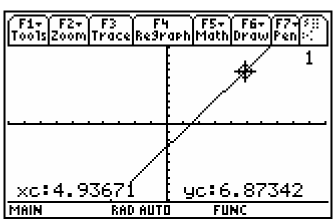


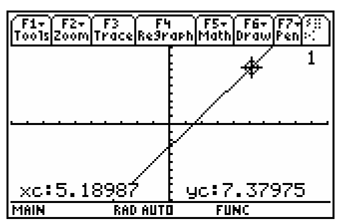
3. Press **[F2] (Zoom) - choice 6** to draw the graph in the standard viewing rectangle.
4. Press **[F3] (Trace).** The screen should now look like:

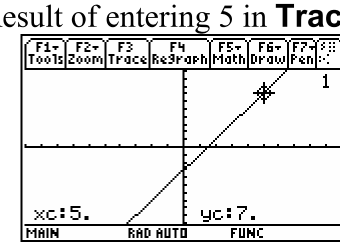


Use the cursor keys to move along the graph, getting "closer and closer" to where $x = 5$, you should see that the y-value gets "closer and closer" to 7. While in **Trace** mode, you may enter a value for x at the keyboard, and press **[ENTER]** to see what happens to y for that exact x-value.

Result of entering 5 in **Trace** mode :

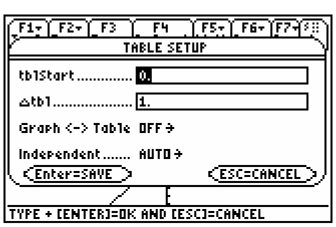


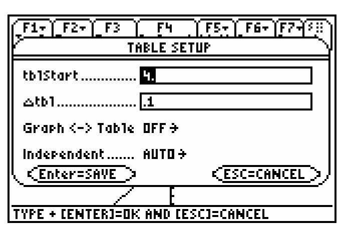




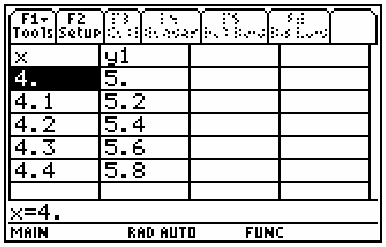
You can also see this in table form by:

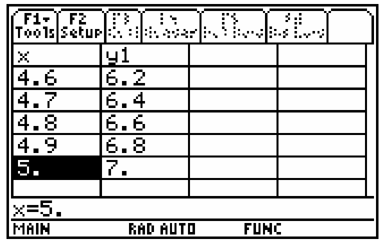
1. Press **[TblSet] ([♦]-[F4])** !!!Do NOT press **[ENTER]** to move from choice to choice!!! Use the cursor keys instead.
Set **table start** = **4**, and **Δtable=.1**. Leave **Graph » tbl** off and **Independent** set to **AUTO**.
Only press **[ENTER]** twice after all changes have been registered.

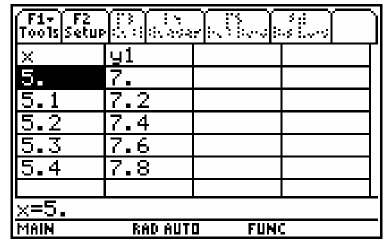




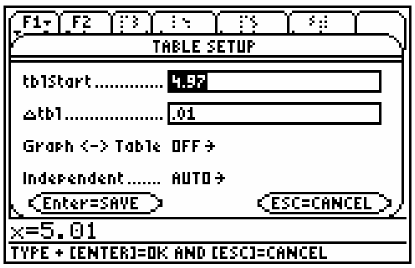
2. Go to **[TABLE] ([♦]-[F5])** and move the cursor down the table. You will see that as you let ***x*** "get closer" to 5, the value of **y1** "gets closer" to 7. Of course, when $x = 5$, **y1** is exactly 7.

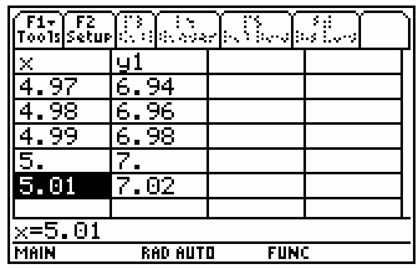






3. Return to the **TblSet** screen and change **Δtbl** to **.01**. Then return to the **TABLE** screen and move up/down in the table to see what $f(x)$ approaches as x approaches 5, this time by hundredths.





In the function $f(x) = 2x - 3$, you can see that as x approaches 5 (gets closer and closer to 5), the value of the function $f(x)$ gets approaches 7 (gets closer and closer to 7).

In symbols, we write: $\lim_{x \to 5}(2x - 3) = 7$

More formally, the ***limit of a function*** as x gets close to (but not necessarily equal to) , or *approaches* some value *a,* is the value (if any) that the function "gets closer and closer to" as x gets "closer and closer (but not necessarily equal) to *a*. If the value the function gets "closer and closer to" is L, we write

Short for limit → lim; *function we are investigating* → $f(x)$

$$\lim_{x \to a} f(x) = L$$

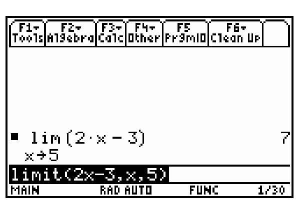
"as x approaches a" → $x \to a$; *the value the function approaches* → L

To find a limit of a function with the TI-89, we use the **limit** command. From the **[HOME]** screen, it is in the **calc menu** (**[F3]**), **choice 3**.

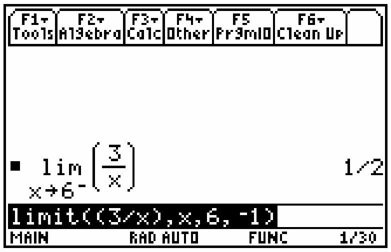
Syntax: **limit(function, variable, value variable approaches, [direction])**

[Direction] is optional. There are only 2 acceptable values for direction:
1: let x approach the value from the right (from the positive direction on the number line,
-1: let x approach the value from the left (from the negative direction on the number line).
No direction number: find the overall limit.

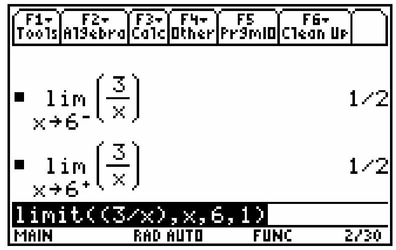
<u>EXAMPLES</u>: Find the limit of $f(x) = 2x - 3$ as x approaches 5, which is symbolically written as $\lim_{x \to 5}(2x - 3)$



Find the left-hand limit as x approaches 6 (from the left): (5.5, 5.6, 5.7, 5.8, 5.9, etc.) of $f(x) = \dfrac{3}{x}$



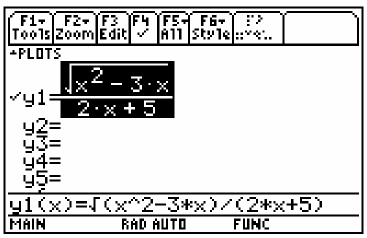
Find the right-hand limit as x approaches 6 (from the right): (6.5, 6.3, 6.1, 6.01, 6.001, etc.) of $f(x) = \dfrac{3}{x}$

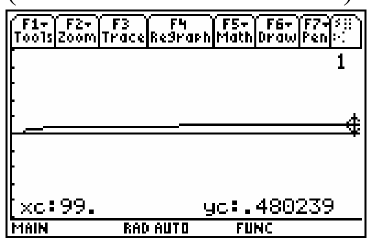


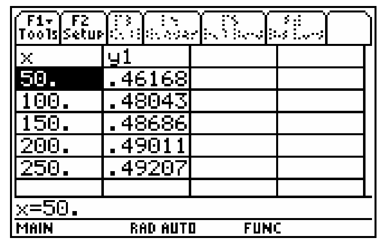
Find the limit as x approaches ∞ of $f(x) = \dfrac{\sqrt{x^2 - 3x}}{2x + 5}$

Graph: **[0,100]×[-5,5], xscl=0** (to turn off scale on x-axis)

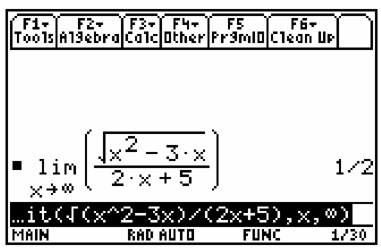
Table





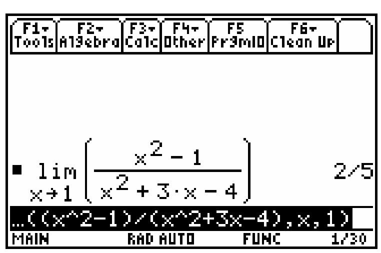


Now calculate $\lim_{x \to \infty} \dfrac{\sqrt{x^2 - 3x}}{2x + 5}$ on the **[HOME]** screen: To obtain **"∞",**use **[♦]- [CATALOG]**



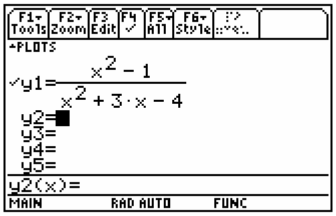
Next, we investigate the function $f(x) = \dfrac{x^2 - 1}{x^2 + 3x - 4}$. If we attempt to calculate $f(1)$, we obtain "$\dfrac{0}{0}$", so $f(1)$ is undefined. (Actually, mathematicians call the $\dfrac{0}{0}$ form "***INDETERMINANT***.")

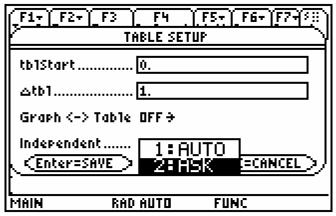
But calculating $\lim_{x \to 1} \dfrac{x^2 - 1}{x^2 + 3x - 4}$:

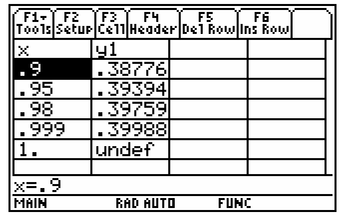


This result may seem somewhat unusual, in light of what we found when we calculated $f(1)$. From that result we might expect that the limit of the function would be undefined, at least.

However, look at the table and the graph: This time, in the **[TblSet]** screen, set Independent to **ASK**. This allows you to type in any x values you should choose, instead of having the calculator generate a table with a constant **Δtbl**. In this way, you can investigate the function as x (the independent variable) gets as close as you want to 1.

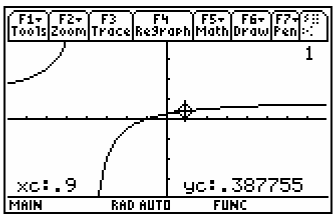




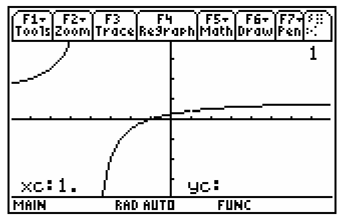


These x-values were entered by hand. Use [←] to delete already completed entries.

Zoom-Decimal Window, **Trace** to $x = .9$



Zoom-Decimal Window, **Trace** to $x = 1$



You will notice that the trace cursor disappears and that there is a tiny gap in the graph at $x = 1$ (see the second graph). Your screen is not malfunctioning. At $x = 1$, the function is undefined, since $f(1) = 0/0$. Therefore there is no point to plot. Consequently the graph does not exist at $x = 1$, and the cursor has no point to show.

From an algebraic standpoint, we can see what is going on.

$$f(x) = \frac{x^2-1}{x^2+3x-4} = \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}(x+3)} = \frac{x+1}{x+4}, \quad \textbf{\textit{provided x} } \boldsymbol{\neq 1,}$$

since then reducing the fraction would be dividing the numerator and denominator by $(1 - 1) = 0$, which is not allowed. Consequently, **as long as $x \neq 1$**, $f(x)$ reduces to $\dfrac{x+1}{x+4}$, and we can use that reduced fraction to calculate $f(x)$. However, when x reaches exactly 1, the original fraction becomes undefined.

If x is merely "close to 1," but not actually 1, the fraction may be reduced because the factor of $(x - 1)$ that is being "cancelled out" *is not zero* (although it will be close to zero). Thus, the function will get close to the value of $\dfrac{x+1}{x+4}$ when $x = 1$, which is 2/5, or .4 . ***The value of*** $\dfrac{x+1}{x+4}$ ***will, however, never be exactly equal to this value.***

This being merely "close to 1" is acceptable, because saying that

$$\lim_{x \to a} f(x) = L$$

means that L is the value the function gets close to as x gets closer and closer to a, but <u>not necessarily equal to a</u>.

Lab 2 NAME(PRINT Clearly!)____________________________LAB TIME_______
Homework

1. Given the function $f(x)=\dfrac{x-5}{x^2-25}$ Use the limit command to:

	COMMAND LINE	RESULT
a. Find the limit as x approaches 5 from the right:	____________________________	________
b. Find the limit as x approaches 5 from the left:	____________________________	________
c. Find the limit as x approaches infinity. (use **[♦]-[CATALOG] to obtain [∞])**	___________________________	________

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2. a. Find $\displaystyle\lim_{x\to 5^+}\frac{x^2+x-30}{x-5}$ **limit((x^2+x-30)/(x-5),x,5,1)** ________

b. Edit the command line to find $\displaystyle\lim_{x\to 5^-}\frac{x^2+x-30}{x-5}$ ________

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3. Numeric (TABLE) investigation of the limit as x approaches 5:

- Go to the **[Y=]** screen **(♦-[F1])**
- Enter $y1=\dfrac{x^2+x-30}{x-5}$.

Go to the **[TblSet] ([♦]-[F4])** screen and set **table start = 4.8** and **Δtable=.01** .
Then go to **the Table ([♦]-[F5])** screen and use the cursor keys to move around in the table and see what happens around and precisely at $x = 5$. The y column is purposely left blank.

You complete the second column of the table:

F1 Tools	F2 Setup			
x				
4.98				
4.99				
5.				
5.01				
5.02				

x=5.
MAIN RAD AUTO FUNC

Describe in words what happens to the value of $f(x)$ as x approaches 5.

What happens to y when x is exactly equal to 5? Why?

Does $\displaystyle\lim_{x\to 5}\frac{x^2+x-30}{x-5}$ exist?

How can this be so, in light of what happens to $f(x)$ when $x =$ 5, exactly?

4. Use the tablefeature to find the limit of the above function $y1 = \dfrac{x^2 + x - 30}{x - 5}$ as x approaches positive infinity:

Set **table start to 50**, and **Δtable to 10**. Move down through the table (or try using the **limit** command) to see what happens to **y1** as **x** gets larger and larger, to find:

a. $\lim\limits_{x \to \infty} \dfrac{x^2 + x - 30}{x - 5}$ a. ________________

b. $\lim\limits_{x \to -\infty} \dfrac{x^2 + x - 30}{x - 5}$ b. ________________

c. press **[♦]-[F2]** to get to the **[WINDOW]** screen and change the window coordinates to:

xmin=-7.9,
xmax=7.9,
xscl=1,
ymin=3,
ymax=15,
yscl=1,
xres=1.

Then press **[GRAPH] ([♦]-[F3])**.
Draw the graph here:

d. Press **[F3] (Trace).** Use the right and left cursor keys to move the "bull's eye" along the graph, and look carefully around where $x = 5$. What does **Y1** get close to when x gets close to 5?

d. __________

e. What happens to the y-value shown when x = 5? e. ________________________

f. Based on the graph, what is $\lim\limits_{x \to 5^-} \dfrac{x^2 + x - 30}{x - 5}$? f. ________

g. What is $\lim\limits_{x \to 5^+} \dfrac{x^2 + x - 30}{x - 5}$? g. _________

h. Find $\lim\limits_{x \to 5} \dfrac{x^2 + x - 30}{x - 5}$ h.________

In 5-6 below, *investigate each function using the table feature*: Enter the function into **Y1.** Then **press [♦]-[F4] ([TblSet])** and set Independent to **ASK**. Press **[ENTER]** twice. Then press **[♦]-[F5] ([TABLE])** and enter several values of x getting closer and closer to and on either side of 1 (In #6a closer to and less than 1, then for 6b, closer to and greater than 1) in order to see what the respective limits are.

5. $\lim\limits_{x \to 1} \dfrac{x+3}{x-1}$ _________ 6a. $\lim\limits_{x \to 1^-} \dfrac{x+3}{x-1}$ _________ 6b. $\lim\limits_{x \to 1^+} \dfrac{x+3}{x-1}$ _________

In 7 and 8 , *investigate each graphically*. Enter the function into **Y1.** Check each in the **Zoom-Decimal window ([♦]-[F2], choice 6)** After the graph is drawn, go to **[WINDOW] ([♦]-[F2]),** and set **xres = 1** Then re-graph and use **trace ([F3])** to investigate. Use the **limit** command ONLY TO CHECK YOUR ANSWER!!!!!!

7. $\lim\limits_{x \to 1^+} \dfrac{x+3}{x-1}$ _________

8. $\lim\limits_{x \to \infty} \dfrac{x+3}{x-1}$ _________
(change **xmin** to **-100** and **xmax** to **100** or even larger values for **xmin** and **xmax**!)

9 – 15 : Find each limit using the **limit** command and give the result.
<u>(Show the command line as it appears in the command line on the screen *after* you have typed it.)</u>

	Command line	result
9. $\lim_{x\to 3}(3x^2-4)$	____	________
10. $\lim_{x\to 2^+}\text{int}(x)$	____	________
11. $\lim_{x\to 2^-}\text{int}(x)$	____	________
12. $\lim_{x\to 2}\text{int}(x)$	____	________
13. $\lim_{x\to\infty}\frac{2x+1}{x-5}$	____	________
14. $\lim_{x\to-\infty}\sqrt{5-x}$	____	________
15. $\lim_{h\to 0}\frac{(5+h)^2-5^2}{h}$	____	________

16 – 17: Find graphical, numeric, and calculator command evidence of the limit, or non-existence of the limit:

16. $\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}$

GRAPHIC: (draw a graph)

NUMERIC (Show the Table you generated)

(Show the command line and result)
COMMAND LINE:

____

RESULT: ____________

17. $\lim_{x\to 0}\frac{(4+x)^2-4^2}{x}$

GRAPHIC: (draw a graph)

NUMERIC (Show the Table you generated)

(Show the command line and the result)
COMMAND LINE:

____

RESULT: ____________